Tilted rotation in nonhydrostatic ocean modelling

Abstract

A linear stability analysis of the inviscid stratified Boussinesq equations is presented given a steady zonal flow with constant vertical shear in a tilted f-plane. Full nonhydrostatic terms are included: 1) acceleration of vertical velocity and 2) Coriolis terms arising from the meridional component of Earth's rotation vector. Calculations of growth rates, critical wavenumbers, and dominance regimes for baroclinic and symmetric instabilities are compared with results from the traditional nonhydrostatic equations, which include a strictly vertical rotation vector, as well as results from the hydrostatic equations. We find that tilted rotation enhances the dominance regime of symmetric instabilities at the expense of baroclinic instabilities and maintains symmetric instabilities to larger scales than previously indicated. Furthermore, in contrast to former studies, we determine that hydrostatic growth rates for both instabilities are not maximal. Rather, growth rates peak in the fully nonhydrostatic equations for parameter regimes physically relevant and consistent with oceanic measurements of the Labrador Sea and Southern Ocean. Results suggest that implementation of the fully nonhydrostatic equations should be considered for high-latitude numerical modelling.

References

[Stone(1971)] Stone, P. H., 1971: Baroclinic stability under non-hydrostatic conditions. *J. Fluid Mech.*, **45**, 659–671.

[Sun(1994)] Sun, W.-Y., 1994: Unsymmetrical symmetric instability. *Q. J. R. Meteorol. Soc.*, **121**, 419–431.

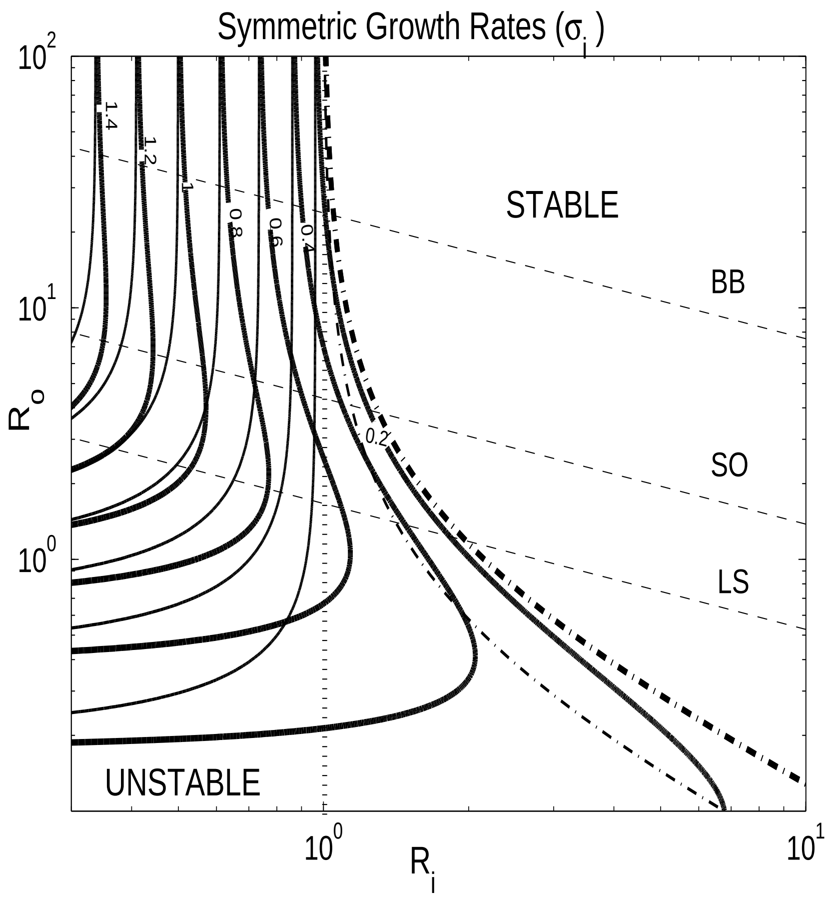

Figure 1: Contours of maximum growth rate for the symmetric instability at $\phi_o = 60^o$. Thick solid lines are solutions which include tilted rotation and Dw/Dt, and thin solid lines are from traditional nonhydrostatic equations ($Dw/Dt \neq 0$ and $\cot\phi_o = 0$). Dashed lines are of constant N typical of the Bay of Biscay (BB), Southern Ocean (SO) and Labrador Sea (LS). The dotted line at $R_i = 1$ marks the boundary for existence of the symmetric instability according to [Stone(1971)]'s necessary condition when $\cot\phi_o = 0$, the thick dash-dotted line defines the existence boundary according to our necessary condition, and the thin dash-dotted line marks the instability criteria of [Sun(1994)]. Symmetric maximum growth rates *exceed* maximum baroclinic rates in regions of negative q except in a small neighborhood of the boundary. This boundary, for LS, reaches $R_i \approx 2$ with tilted rotation and Dw/Dt, nearly double that of the traditional nonhydrostatic and hydrostatic results.

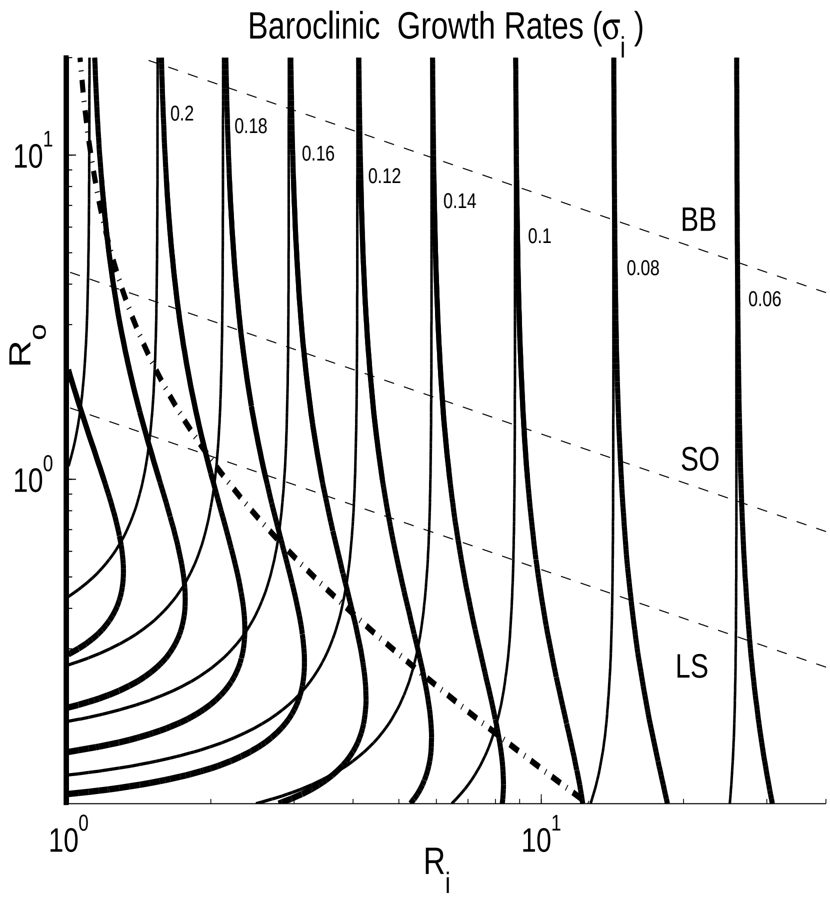

Figure 2: Contours of maximum growth rate for the baroclinic instability at $\phi_o = 60^o$. Thick solid lines are solutions which include tilted rotation and Dw/Dt, and thin solid lines are from traditional nonhydrostatic equations ($Dw/Dt \neq 0$ and $\cot\phi_o = 0$). Dashed lines are of constant N typical of the Bay of Biscay (BB), Southern Ocean (SO) and Labrador Sea (LS). Regimes of negative q, for which the symmetric instability occurs and dominates except very near the boundary, are indicated by the parameter space to the left of the thick dash-dotted line, when full nonhydrostatic effects are included, and by the vertical axis $R_i = 1$, when either a hydrostatic model or [Stone(1971)]'s traditional nonhydrostatic model is assumed. Inclusion of tilted rotation in the nonhydrostatic model reduces the parameter space for which baroclinic instabilities dominate, and this effect is most pronounced for stratifications typical of the high-latitude regions. For further details see figure 1